OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4756

Further Methods for Advanced Mathematics (FP2)

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet
  (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Monday 20 June 2011
Morning

Duration: 1 hour 30 minutes

* 4 7 5 6 *

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (54 marks)

Answer all the questions

1 **(a)** A curve has polar equation $r = a(1 - \sin\theta)$, where $a > 0$ and $0 \leqslant \theta < 2\pi$.

(i) Sketch the curve. **[2]**

(ii) Find, in an exact form, the area of the region enclosed by the curve. **[7]**

(b) **(i)** Find, in an exact form, the value of the integral $\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{1+4x^2}\,\mathrm{d}x$. **[3]**

(ii) Find, in an exact form, the value of the integral $\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\left(1+4x^2\right)^{\frac{3}{2}}}\,\mathrm{d}x$. **[6]**

2 **(a)** Use de Moivre's theorem to find expressions for $\sin 5\theta$ and $\cos 5\theta$ in terms of $\sin\theta$ and $\cos\theta$.

Hence show that, if $t = \tan\theta$, then

$$\tan 5\theta = \frac{t(t^4 - 10t^2 + 5)}{5t^4 - 10t^2 + 1}.$$ **[6]**

(b) **(i)** Find the 5th roots of $-4\sqrt{2}$ in the form $r\mathrm{e}^{\mathrm{j}\theta}$, where $r > 0$ and $0 \leqslant \theta < 2\pi$. **[4]**

These 5th roots are represented in the Argand diagram, in order of increasing θ, by the points A, B, C, D, E.

(ii) Draw the Argand diagram, making clear which point is which. **[2]**

The mid-point of AB is the point P which represents the complex number w.

(iii) Find, in exact form, the modulus and argument of w. **[3]**

(iv) w is an nth root of a real number a, where n is a positive integer. State the least possible value of n and find the corresponding value of a. **[3]**

3 **(i)** Find the value of k for which the matrix

$$\mathbf{M} = \begin{pmatrix} 1 & -1 & k \\ 5 & 4 & 6 \\ 3 & 2 & 4 \end{pmatrix}$$

does not have an inverse.

Assuming that k does not take this value, find the inverse of $\mathbf{M}$ in terms of k. **[7]**

(ii) In the case $k = 3$, evaluate

$$\mathbf{M} \begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}.$$ **[2]**

(iii) State the significance of what you have found in part **(ii)**. **[2]**

(iv) Find the value of t for which the system of equations

$$\begin{aligned} x - \ \ y + 3z &= t \\ 5x + 4y + 6z &= 1 \\ 3x + 2y + 4z &= 0 \end{aligned}$$

has solutions. Find the general solution in this case and describe the solution geometrically. **[7]**

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Given that $\cosh y = x$, show that $y = \pm \ln\left(x + \sqrt{x^2 - 1}\right)$ and that $\operatorname{arcosh} x = \ln\left(x + \sqrt{x^2 - 1}\right)$. **[7]**

(ii) Find $\displaystyle\int_{\frac{4}{5}}^{1} \frac{1}{\sqrt{25x^2 - 16}}\,\mathrm{d}x$, expressing your answer in an exact logarithmic form. **[5]**

(iii) Solve the equation

$$5\cosh x - \cosh 2x = 3,$$

giving your answers in an exact logarithmic form. **[6]**

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 In this question, you are required to investigate the curve with equation

$$y = x^m(1-x)^n, \qquad 0 \leqslant x \leqslant 1,$$

for various positive values of m and n.

(i) On separate diagrams, sketch the curve in each of the following cases.

(A) $m = 1, n = 1$,

(B) $m = 2, n = 2$,

(C) $m = 2, n = 4$,

(D) $m = 4, n = 2$. **[4]**

(ii) What feature does the curve have when $m = n$?

What is the effect on the curve of interchanging m and n when $m \neq n$? **[2]**

(iii) Describe how the x-coordinate of the maximum on the curve varies as m and n vary. Use calculus to determine the x-coordinate of the maximum. **[6]**

(iv) Find the condition on m for the gradient to be zero when $x = 0$. State a corresponding result for the gradient to be zero when $x = 1$. **[2]**

(v) Use your calculator to investigate the shape of the curve for large values of m and n. Hence conjecture what happens to the value of the integral $\int_0^1 x^m(1-x)^n \, dx$ as m and n tend to infinity. **[2]**

(vi) Use your calculator to investigate the shape of the curve for small values of m and n. Hence conjecture what happens to the shape of the curve as m and n tend to zero. **[2]**

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4756:** Further Methods for Advanced Mathematics

Mark Scheme for June 2011

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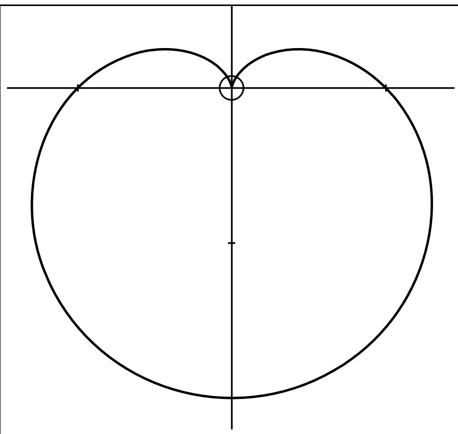
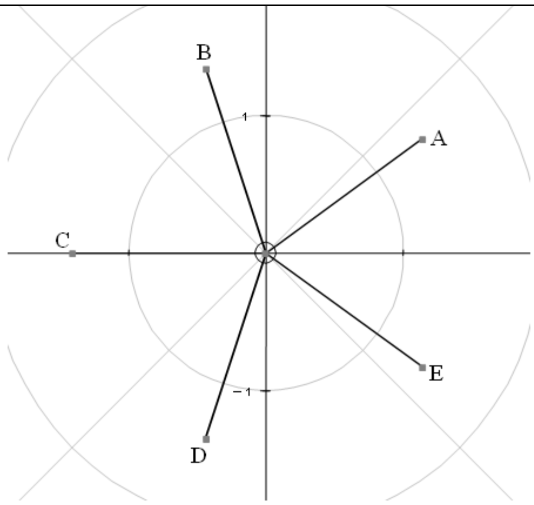
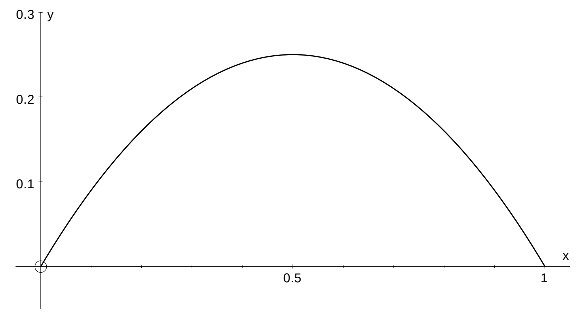
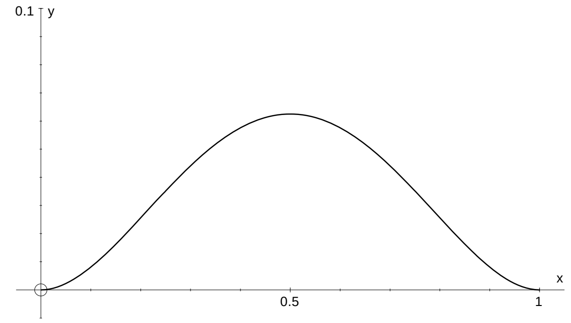
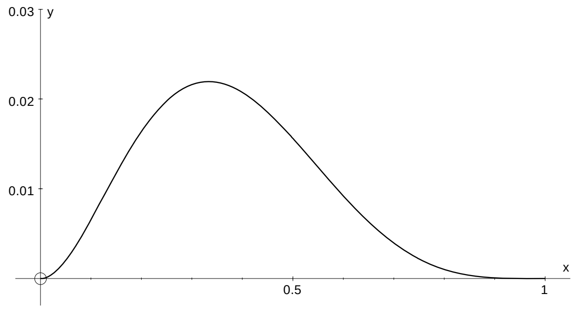
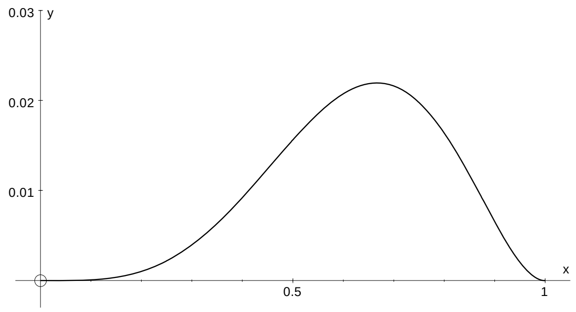
4756 (FP2) Further Methods for Advanced Mathematics

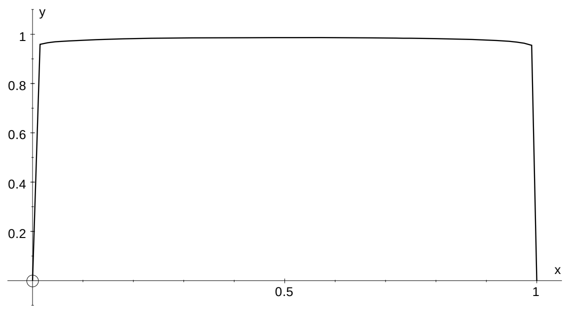
1 (a)(i)		G1 G1	2	Correct general shape including symmetry in vertical axis Correct form at O and no extra sections. Dependent on first G1 For an otherwise correct curve with a sharp point at the bottom, award G1G0
(ii)	$\text{Area} = \frac{1}{2}a^2\int_0^{2\pi}(1-\sin\theta)^2\,d\theta$ $= \frac{1}{2}a^2\int_0^{2\pi}(1-2\sin\theta+\sin^2\theta)\,d\theta$ $= \frac{1}{2}a^2\int_0^{2\pi}\left(\frac{3}{2}-2\sin\theta-\frac{1}{2}\cos 2\theta\right)d\theta$ $= \frac{1}{2}a^2\left[\frac{3}{2}\theta+2\cos\theta-\frac{1}{4}\sin 2\theta\right]_0^{2\pi}$ $= \frac{3}{2}\pi a^2$	M1 M1 A1 M1 A2 A1	7	Integral expression involving $(1-\sin\theta)^2$ Expanding Correct integral expression, incl. limits (which may be implied by later work) Using $\sin^2\theta = \frac{1}{2}-\frac{1}{2}\cos 2\theta$ Correct result of integration. Give A1 for one error Dependent on previous A2
(b)(i)	$\int_{-\frac{1}{2}}^{\frac{1}{2}}\frac{1}{1+4x^2}\,dx = \frac{1}{4}\int_{-\frac{1}{2}}^{\frac{1}{2}}\frac{1}{\frac{1}{4}+x^2}\,dx = \frac{1}{4}\left[2\arctan 2x\right]_{-\frac{1}{2}}^{\frac{1}{2}}$ $= \frac{1}{2}\left(\frac{\pi}{4}-\left(-\frac{\pi}{4}\right)\right)$ $= \frac{\pi}{4}$	M1 A1 A1	3	arctan alone, or any tan substitution $\frac{1}{4}\times 2$ and $2x$ Evaluated in terms of π
(ii)	$x = \frac{1}{2}\tan\theta$ $\Rightarrow dx = \frac{1}{2}\sec^2\theta\,d\theta$ $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1}{(\sec^2\theta)^{\frac{3}{2}}}\times\frac{\sec^2\theta}{2}\,d\theta$ $= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{1}{2}\cos\theta\,d\theta$ $= \left[\frac{1}{2}\sin\theta\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$ $= \frac{1}{2}\left(\frac{1}{\sqrt{2}}-\left(-\frac{1}{\sqrt{2}}\right)\right)$ $= \frac{1}{\sqrt{2}}$	M1 A1A1 M1 A1ft A1	6	Any tan substitution $\frac{1}{(\sec^2\theta)^{\frac{3}{2}}}$, $\frac{\sec^2\theta}{2}$ Integrating $a\cos b\theta$ and using consistent limits. Dependent on M1 above $\frac{a}{b}\sin b\theta$
				18

2 (a)	$\cos 5\theta + j\sin 5\theta = (\cos\theta + j\sin\theta)^5$ $= c^5 + 5c^4js - 10c^3s^2 - 10c^2js^3 + 5cs^4 + js^5$	M1 M1	Expanding Separating real and imaginary parts. Dependent on first M1
	$\Rightarrow \cos 5\theta = c^5 - 10c^3s^2 + 5cs^4$ $\sin 5\theta = 5c^4s - 10c^2s^3 + s^5$	A1 A1	Alternative: $16c^5 - 20c^3 + 5c$ Alternative: $16s^5 - 20s^3 + 5s$
	$\Rightarrow \tan 5\theta = \dfrac{5c^4s - 10c^2s^3 + s^5}{c^5 - 10c^3s^2 + 5cs^4}$ $= \dfrac{5t - 10t^3 + t^5}{1 - 10t^2 + 5t^4}$	M1	Using $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and simplifying
	$= \dfrac{t\left(t^4 - 10t^2 + 5\right)}{5t^4 - 10t^2 + 1}$	A1 (ag) **6**	
(b)(i)	$\arg(-4\sqrt{2}) = \pi$ $\Rightarrow$ fifth roots have $r = \sqrt{2}$	B1	
	and $\theta = \dfrac{\pi}{5}$	B1	No credit for arguments in degrees
	$\Rightarrow z = \sqrt{2}e^{\frac{1}{5}j\pi}, \sqrt{2}e^{\frac{3}{5}j\pi}, \sqrt{2}e^{j\pi}, \sqrt{2}e^{\frac{7}{5}j\pi}, \sqrt{2}e^{\frac{9}{5}j\pi}$	M1 A1 **4**	Adding (or subtracting) $\dfrac{2\pi}{5}$ All correct. Allow $-\pi \le \theta < \pi$
(ii)		G1 G1 **2**	Points at vertices of "regular" pentagon, with one on negative real axis Points correctly labelled
(iii)	$\arg(w) = \dfrac{1}{2}\left(\dfrac{\pi}{5} + \dfrac{3\pi}{5}\right) = \dfrac{2\pi}{5}$	B1	
	$\lvert w\rvert = \sqrt{2}\cos\dfrac{\pi}{5}$	M1 A1ft **3**	Attempting to find length F.t. (positive) r from (i)
(iv)	$w = \sqrt{2}\cos\dfrac{\pi}{5}\, e^{\frac{2}{5}\pi j} \Rightarrow w^n = \left(\sqrt{2}\cos\dfrac{\pi}{5}\right)^n e^{\frac{2}{5}\pi nj}$		
	which is real if $\sin\dfrac{2\pi n}{5} = 0 \Rightarrow n = 5$	B1	
	$\lvert w^5\rvert = \left(\sqrt{2}\cos\dfrac{\pi}{5}\right)^5$	M1	Attempting the nth power of his modulus in (iii), or attempting the modulus of the nth power here
	$\Rightarrow a = 2^{\frac{5}{2}}\cos^5\dfrac{\pi}{5}$	A1 **3**	Accept 1.96 or better **18**

3 (i)	$\det(\mathbf{M}) = 1(16-12) + 1(20-18) + k(10-12)$ $= 6 - 2k$ $\Rightarrow$ no inverse if $k = 3$ $\mathbf{M}^{-1} = \frac{1}{6-2k}\begin{pmatrix} 4 & 4+2k & -6-4k \\ -2 & 4-3k & 5k-6 \\ -2 & -5 & 9 \end{pmatrix}$	M1 A1 A1 M1 A1 M1 A1 **7**	Obtaining det(**M**) in terms of k Accept $k \neq 3$ after correct determinant Evaluating at least four cofactors (including one involving k) Six signed cofactors correct (including one involving k) Transposing and dividing by det(**M**). Dependent on previous M1M1
(ii)	$\begin{pmatrix} 1 & -1 & 3 \\ 5 & 4 & 6 \\ 3 & 2 & 4 \end{pmatrix}\begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}$	M1 A1 **2**	Setting $k = 3$ and multiplying
(iii)	$\begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}$ is an eigenvector corresponding to an eigenvalue of 1	B1 B1 **2**	For credit here, 2/2 scored in (ii) Accept "invariant point"
(iv)	$3x + 6y = 1 - 2t$, $x + 2y = 2$, $5x + 10y = -4t$ (or $9x + 18z = 4t + 1$, $5x + 10z = 2t$, $x + 2z = -1$) (or $9y - 9z = 1 - 5t$, $5y - 5z = -3t$, $2y - 2z = 3$) For solutions, $1 - 2t = 3 \times 2$ $\Rightarrow t = -\frac{5}{2}$ $x = \lambda$, $y = 1 - \frac{1}{2}\lambda$, $z = -\frac{1}{2} - \frac{1}{2}\lambda$ Straight line	M1 A1 M1 A1 M1 A1 B1 **7**	Eliminating one variable in two different ways Two correct equations Validly obtaining a value of t Obtaining general solution by setting one unknown = λ and finding other two in terms of λ (accept unknown instead of λ) Accept "sheaf". Independent of all previous marks **18**

4 (i)	$\cosh y = x \Rightarrow x = \frac{1}{2}\left(e^{y} + e^{-y}\right)$	B1	Using correct exponential definition
	$\Rightarrow 2x = e^{y} + e^{-y}$		
	$\Rightarrow \left(e^{y}\right)^{2} - 2xe^{y} + 1 = 0$	M1	Obtaining quadratic in e^{y}
	$\Rightarrow e^{y} = \frac{2x \pm \sqrt{4x^2 - 4}}{2} = x \pm \sqrt{x^2 - 1}$	M1 A1	Solving quadratic $x \pm \sqrt{x^2 - 1}$
	$\Rightarrow y = \ln(x \pm \sqrt{x^2 - 1})$		
	$\left(x + \sqrt{x^2 - 1}\right)\left(x - \sqrt{x^2 - 1}\right) = 1$	M1	Validly attempting to justify ± in printed answer
	$\Rightarrow y = \pm\ln(x + \sqrt{x^2 - 1})$	A1 (ag)	
	arcosh(x) = $\ln(x + \sqrt{x^2 - 1})$ because this is the principal value	B1	Reference to arcosh as a function, or correctly to domains/ranges
		7	
(ii)	$\int_{\frac{4}{5}}^{1} \frac{1}{\sqrt{25x^2 - 16}}\,dx = \frac{1}{5}\int_{\frac{4}{5}}^{1} \frac{1}{\sqrt{x^2 - \frac{16}{25}}}\,dx$		
	$= \frac{1}{5}\left[\text{arcosh}\left(\frac{5x}{4}\right)\right]_{\frac{4}{5}}^{1}$	M1 A1A1	arcosh alone, or any cosh substitution $\frac{1}{5}, \frac{5x}{4}$
	$= \frac{1}{5}\left(\text{arcosh}\left(\frac{5}{4}\right) - \text{arcosh}(1)\right)$		
	$= \frac{1}{5}\ln\left(\frac{5}{4} + \sqrt{\left(\frac{5}{4}\right)^2 - 1}\right) - 0$	M1	Substituting limits and using (i) correctly at any stage (or using limits of u in logarithmic form). Dep. on first M1
	$= \frac{1}{5}\ln 2$	A1	
	OR $= \frac{1}{5}\left[\ln\left(x + \sqrt{x^2 - \frac{16}{25}}\right)\right]_{\frac{4}{5}}^{1}$ M A1A $= \frac{1}{5}\ln\frac{8}{5} - \frac{1}{5}\ln\frac{4}{5}$ $= \frac{1}{5}\ln 2$ A		$\ln\left(kx + \sqrt{k^2x^2 + \ldots}\right)$ Give M1 for $\ln\left(k_1x + \sqrt{k_2{}^2x^2 + \ldots}\right)$ $\frac{1}{5}$, $\ln\left(x + \sqrt{x^2 - \frac{16}{25}}\right)$ o.e.
		5	
(iii)	$5\cosh x - \cosh 2x = 3$		
	$\Rightarrow 5\cosh x - (2\cosh^2 x - 1) = 3$	M1	Attempting to express $\cosh 2x$ in terms of $\cosh x$
	$\Rightarrow 2\cosh^2 x - 5\cosh x + 2 = 0$		
	$\Rightarrow (2\cosh x - 1)(\cosh x - 2) = 0$	M1	Solving quadratic to obtain at least one real value of $\cosh x$
	$\Rightarrow \cosh x = \frac{1}{2}$ (rejected)	A1	Or factor $2\cosh x - 1$
	or $\cosh x = 2$	A1	
	$\Rightarrow x = \ln\left(2 + \sqrt{3}\right)$	A1ft	F.t. $\cosh x = k$, $k > 1$
	$x = -\ln\left(2 + \sqrt{3}\right)$ or $\ln\left(2 - \sqrt{3}\right)$	A1ft **6**	F.t. other value. Max. A1A0 if additional real values quoted **18**

5 (i)	**(A)** $m = 1, n = 1$ 	G1	Negative parabola from (0,0) to (1,0), symmetrical about $x = 0.5$
	(B) $m = 2, n = 2$ 	G1	Bell-shape from (0,0) to (1,0), symmetrical about $x = 0.5$; flat ends, and obviously different to (A)
	(C) $m = 2, n = 4$ 	G1	Skewed curve from (0,0) to (1,0), maximum to left of $x = 0.5$
	(D) $m = 4, n = 2$ 	G1 **4**	Skewed curve from (0,0) to (1,0), maximum to right of $x = 0.5$
(ii)	When $m = n$, the curve is symmetrical Exchanging m and n reflects the curve	B1 B1 **2**	
(iii)	If $m > n$, the maximum is to the right of $x = 0.5$ As m increases relative to n, the maximum point moves further to the right $y = x^m(1-x)^n \Rightarrow \dfrac{dy}{dx} = mx^{m-1}(1-x)^n - nx^m(1-x)^{n-1}$ $= x^{m-1}(1-x)^{n-1}\left[m(1-x) - nx\right]$ $\dfrac{dy}{dx} = 0 \Rightarrow$ maximum at $x = \dfrac{m}{m+n}$	B1 B1 M1 A1 M1 A1 **6**	o.e. Give B1B0 if the idea is correct but vaguely expressed Using product rule Any correct form Setting derivative = 0 and solving to find a value of x other than 0 or 1

(iv)	$y'(0) = 0$ provided $m > 1$ $y'(1) = 0$ provided $n > 1$	B1 B1 **2**	
(v)	For large m and n, the curve approaches the x-axis $\Rightarrow \int_0^1 x^m (1-x)^n dx \to 0$ as $m, n \to \infty$	B1 B1 **2**	Comment on shape Independent
(vi)	e.g. $m = 0.01$, $n = 0.01$  The curve tends to $y = 1$	M1 A1 **2**	Evidence of investigation s.o.i. Accept "three sides of (unit) square" **18**

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4756: Further Methods for Advanced Mathematics

General Comments

The total number of candidates was about the same as in Summer 2010, and the overall standard of work was also about the same. Some candidates were very well prepared for the paper and scored highly throughout; about 20% scored 60 marks or more. Others appeared well prepared for some of the paper, scoring highly on one or two questions but achieving much lower marks on the other questions. These questions varied, so there was little difference between the means: Questions 3 and 4 were the best done by a small margin, with Question 2 scoring the lowest. Only a very small minority of candidates seemed to be unprepared, with about 5% scoring fewer than 20 marks. Few candidates attempted Question 5.

Very few candidates appeared to run out of time, although inefficient methods often caused much waste of time and paper, especially in Question 3 (iv). Most candidates' work was presented coherently and legibly. From January 2012, this paper will have a printed answer book.

Comments on Individual Questions

1 (Polar curves, calculus with trigonometric functions)

About a fifth of candidates found this a most agreeable start and achieved full marks. Throughout the question, a very small number of candidates appeared unaware of the need to use radians when integrating with trigonometric functions.

(a) Most candidates were able to score both marks for the sketch of the cardioid in part (i), although some curves could have been more obviously symmetric, and others made the shape pointed at the bottom. A sketch which itself earns marks should be completed more carefully than a sketch which, for example, just aids the solution of an inequality.

For the area in part (ii), the vast majority of candidates knew what to do, but often failed to score all seven marks through inaccuracies such as losing the ½ or a^2 at some point, using an incorrect double angle formula to substitute for $\sin^2\theta$, using incorrect limits, or even failing to expand $(1 - \sin\theta)^2$ correctly. A small minority of candidates seemed unaware of how to deal with the integration of $\sin^2\theta$, giving as their result $\frac{1}{3}\sin^3\theta$ or similar.

(b) Part (i) was done well, with many candidates able to write down the result of the integration immediately, although a number omitted the factor of ½.

Part (ii) discriminated well. Most of those who began with the correct substitution were able to score most of the marks. It was interesting to see a few candidates successfully using the substitution $x = \frac{1}{2}\sinh u$. Far more common were substitutions which led nowhere, e.g. $u = 1 + 4x^2$. A small number of candidates gave the correct exact answer with no working: these had obviously used their calculators to perform the integration. No credit was given for this approach.

2 (Complex numbers)

A surprisingly large number of candidates attempted the question but scored no marks at all.

(a) The majority of candidates were able to score the first four marks in this part. They were able to expand $(\cos\theta + j\sin\theta)^5$ correctly and separate the real and imaginary parts, although a few were determined to use the $z \pm z^{-1}$ method. A significant proportion then went on to do substantial work to express $\cos 5\theta$ in terms of $\cos\theta$ and $\sin 5\theta$ in terms of $\sin\theta$. This attracted no further credit and had the unfortunate effect of making the demonstration of the given expression for $\tan 5\theta$ much more difficult, with those who took this approach having to use the identity $1 + \tan^2\theta = \sec^2\theta$.

(b) Candidates who recognised that the modulus and argument of $-4\sqrt{2}$ were $4\sqrt{2}$ and π respectively had little trouble with parts (i) and (ii). However, a substantial number gave the modulus as $-4\sqrt{2}$, and the argument as 0, which lost them a significant number of marks. The question did ask for the fifth roots in the form $re^{j\theta}$ with $r > 0$ and $0 \le \theta < 2\pi$. Arguments in the range $-\pi$ to π were also accepted.

In part (iii), most candidates were able to give the argument of *w* correctly, but relatively few managed to connect the modulus with a length which could be found by elementary trigonometry. This length was sometimes negative. The value of *n* was often correct in part (iv), but a correct value of *a* was rare.

3 (Matrices and simultaneous equations)

Part (iv) was the discriminator: many candidates managed most of the first 11 marks without too much trouble.

(i) Candidates usually manage to find the determinant and inverse of a 3 × 3 matrix with little difficulty, and this series was no exception. Some interesting methods were seen, such as using a scalar triple product to find the determinant and the vector products of columns of **M**, but most candidates stuck to the more familiar method in which they appeared well versed. There were, as usual, slips in some of the cofactors. A few candidates, having obtained the determinant correctly as $6 - 2k$, "simplified" it to $3 - k$.

(ii) Multiplying **M** by a column vector presented very few problems.

(iii) Again, this part presented few problems, with the vast majority of candidates noticing that the column vector in part (ii) represented an invariant point (which was frequently expressed as e.g. "transforms to itself") or was an eigenvector with eigenvalue 1. The last of these observations was occasionally omitted.

(iv) A variety of approaches was possible here. The majority of candidates began by eliminating unknowns, and a high proportion made elementary errors trying to do this: they would have done better to write out all their working, rather than trying to do it mentally. Errors here often led to candidates substituting incorrect expressions into other expressions, which in turn led to a downward spiral which wasted a great deal of time. An alternative approach which was sometimes taken was to find a point on planes (2) and (3) and substitute it into plane (1): this found the value of t efficiently, but candidates could not always develop this method to find the general solution, which most did recognise as a line, or "sheaf" which was also accepted. Some candidates were determined to use the inverse matrix, even though they also correctly stated that the solution was not unique. Having identified one eigenvector and eigenvalue earlier in the question, a few candidates were determined to use this part to find the other eigenvalues (and, sometimes, eigenvectors): this was not required and gained no credit.

4 (Hyperbolic functions)

(i) Most candidates were able to score the first four marks here by deriving a quadratic equation in e^y and giving both roots, although one or two candidates just stated that 'the result appears in *Examination Formulae and Tables*, so it must be true'. The remaining three marks were gained much less frequently. The majority of candidates took the ± sign outside the log expression without an attempt to explain why, and the explanation of why we take arcosh x as the positive root was often spurious.

(ii) This part was done well. As in Question 1(b), many candidates were able to write down the result of the integration immediately, using arcosh or the logarithmic form given in *Examination Formulae and Tables*. A number even went straight to an expression involving $\ln(5x+\sqrt{25x^2-16})$. The most common error was in dealing with the factor of $\frac{1}{5}$; many missed it out entirely, or gave $\frac{1}{4}$ or $\frac{4}{5}$ or $\frac{5}{4}$.

(iii) This part was also done well. Most candidates substituted for cosh $2x$, solved the resulting quadratic and used part (i) or an equivalent method to obtain the root $\ln(2+\sqrt{3})$. This scored five marks out of six. The other root, $-\ln(2+\sqrt{3})$, was seen much less frequently. A number of candidates used an incorrect substitution for cosh $2x$, while others changed everything to exponentials: this resulted in a quartic equation in e^x, which candidates had to factorise into two quadratic factors to gain any credit. A few managed this, but correct answers by this route were very rare.

5 (Investigations of Curves)

Fewer than 2% of candidates answered this question, and their attempts were often fragmentary: only a handful scored 10 marks or more.

(i) Candidates often ignored the instruction to consider values of x between 0 and 1 and used the default setting for their x values on their graphical calculators. The result was that it was very difficult to see differences between their four curves. We expect sketches to be clear enough to show important features.

(ii) Most were able to say that the graph was symmetric when $m = n$ and could describe the effect of interchanging m and n.

(iii) The description of how the maximum varied was rarely sufficiently precise. Most could use the product rule to differentiate, but only a few were able to obtain the value $\frac{m}{m+n}$.

(iv) Very few candidates attracted credit here. Most asserted that *m* and/or *n* should be zero, ignoring the fact that the question states that they are positive.

(v) Most were able to see that the curve approached the *x*-axis and (especially) that the integral tends to zero.

(vi) There were some appropriate sketches and descriptions.

GCE Mathematics (MEI)

			Max Mark	a	b	c	d	e	u
4751/01	(C1) MEI Introduction to Advanced Mathematics	Raw	72	55	49	43	37	32	0
		UMS	100	80	70	60	50	40	0
4752/01	(C2) MEI Concepts for Advanced Mathematics	Raw	72	53	46	39	33	27	0
		UMS	100	80	70	60	50	40	0
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	48	42	36	29	0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics	Raw	90	63	56	50	44	38	0
		UMS	100	80	70	60	50	40	0
4755/01	(FP1) MEI Further Concepts for Advanced Mathematics	Raw	72	59	52	45	39	33	0
		UMS	100	80	70	60	50	40	0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics	Raw	72	55	48	42	36	30	0
		UMS	100	80	70	60	50	40	0
4758/01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	57	51	45	39	0
4758/02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758/82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
4758	(DE) MEI Differential Equations with Coursework	UMS	100	80	70	60	50	40	0
4761/01	(M1) MEI Mechanics 1	Raw	72	60	52	44	36	28	0
		UMS	100	80	70	60	50	40	0
4762/01	(M2) MEI Mechanics 2	Raw	72	64	57	51	45	39	0
		UMS	100	80	70	60	50	40	0
4763/01	(M3) MEI Mechanics 3	Raw	72	59	51	43	35	27	0
		UMS	100	80	70	60	50	40	0
4764/01	(M4) MEI Mechanics 4	Raw	72	54	47	40	33	26	0
		UMS	100	80	70	60	50	40	0
4766/01	(S1) MEI Statistics 1	Raw	72	53	45	38	31	24	0
		UMS	100	80	70	60	50	40	0
4767/01	(S2) MEI Statistics 2	Raw	72	60	53	46	39	33	0
		UMS	100	80	70	60	50	40	0
4768/01	(S3) MEI Statistics 3	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4769/01	(S4) MEI Statistics 4	Raw	72	56	49	42	35	28	0
		UMS	100	80	70	60	50	40	0
4771/01	(D1) MEI Decision Mathematics 1	Raw	72	51	45	39	33	27	0
		UMS	100	80	70	60	50	40	0
4772/01	(D2) MEI Decision Mathematics 2	Raw	72	58	53	48	43	39	0
		UMS	100	80	70	60	50	40	0
4773/01	(DC) MEI Decision Mathematics Computation	Raw	72	46	40	34	29	24	0
		UMS	100	80	70	60	50	40	0
4776/01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	62	55	49	43	36	0
4776/02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776/82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
4776	(NM) MEI Numerical Methods with Coursework	UMS	100	80	70	60	50	40	0
4777/01	(NC) MEI Numerical Computation	Raw	72	55	47	39	32	25	0
		UMS	100	80	70	60	50	40	0